Final Exam Review Session - Problems

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1 Integrals

Problem 1: Using <u>the definition</u> of the integral, evaluate:

$$\int_0^1 \left(x^2 - 1\right) dx$$

You may use the following formulas:

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Problem 2: Evaluate the following limit:

$$\lim_{n\to\infty} \frac{1^4}{n^5} + \frac{2^4}{n^5} + \cdots + \frac{n^4}{n^5}$$

Problem 3: Evaluate the following integrals

(a) $\int_0^1 x - \frac{1}{\sqrt{x}} + x\sqrt{x} dx$

(b) $\int_{-5}^5 \sqrt{25 - x^2} dx$

(c) $\int_{-1}^1 1 - |x| \, dx$

(d) $\int \left(1 + \tan(\theta)\right)^5 \sec^2(\theta) d\theta$

(e) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(f) The derivative of $g(x) = \int_{\tan(x)}^{\ln(x)} \frac{1}{\ln(t)} dt$

(g) $\int_{-2\pi}^{2\pi} \left(\sin\left(x^3\right)\right)^{2011} \left(\cos\left(e^{x^2}\right)\left(x^{42}+1\right)\right) dx$

(h) An antiderivative F of $f(x) = \frac{1}{x^2+1} + \frac{1}{\sqrt{1-x^2}}$ which satisfies $F(0) = 1$.

(i) The average value of $f(x) = \sin(x)\cos(x)^4$ over $[0, \pi]$

Problem 4:

Find the area of the region enclosed by $y = x^2 - 2x$ and $y = x + 4$

2 Limits

Problem 5: Evaluate the following limits

(a) $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$

(b) $\lim_{x\to 0} (1-2x)^{\frac{1}{x}}$

(c) $\lim_{x\to 0^+} \sqrt{x} e^{\sin\left(\frac{1}{x}\right)}$

(d) $\lim_{x\to\infty} \frac{\sqrt{x^4+2x}}{x^2}$

3 Derivatives / Max/Min

Problem 6 Find the derivatives of the following functions:

(a) $f(x) = \ln(\ln(\ln(\ln(x))))$

(b) $f(x) = x^{\sin(x)}$

(c) y', where $x^2 + xy + y^2 = \sin(y)$

(d) y' at $\left(0, \frac{1}{2}\right)$, where $x^2 + y^2 = \left(2x^2 + 2y^2 - x\right)^2$

Problem 7

Find the absolute max/min of $f(x) = x^3 - 3x + 1$ on $[0, 3]$

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Problem 8

If $f(0) = 0$ and $f'(x) \geq 1$ for $0 \leq x \leq 1$, how small can $f(1)$ possibly be?

Problem 9

Show $2x - 1 - \sin(x) = 0$ has at most one zero.

Problem 10

A ladder 5 feet long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the angle between the ladder and the ground changing when the bottom is 4 feet from the wall?

Problem 11

A farmer wants to fence an area of 600 square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. Find the dimensions of the fence that minimize the cost of building the fence?